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# *MEASURING THE MENTAL STRENGTH OF AN ARMY*

By Major Robert M. Yerkes
Sanitary Corps, N. A.

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Committees of the American Psychological Association and the Psychology Committee of the National Research Council prepared between April and August, 1917, various psychological methods for the selection and classification of recruits in the Army and Navy. In August, 1917, the Surgeon General on recommendation of Majors Victor C. Vaughan and William H. Welch of the National Research Council accepted for trial certain methods which had been devised especially for the psychological examination of men enlisted in the United States Army. Dr. Robert M. Yerkes, Chairman of the Psychology Committee of the National Research Council was appointed on August 17, Major in the Sanitary Corps, to organize and direct this new branch of the service for the Medical Department.

During the initial development of this service, Major Yerkes worked under the immediate administrative supervision of Major Pearce Bailey, Chief of the Division of Neurology, Psychiatry and Psychology. Plans were speedily prepared and the necessary authorization secured from the Secretary of War for thorough trial of the proposed methods in four National Army cantonments. For this work sixteen psychologists were commissioned first lieutenants in the Sanitary Corps, and twenty-four others were given civil appointment by special authorization of the Secretary of War. The psychological staff of each camp consisted of ten men.

Between October 1 and December 1, 1917, nearly 100,000 drafted men, students in Officers' Training Camps, and officers of camps or divisions were examined. In December the work was officially inspected by an officer of the Medical Corps. The reports of this inspectional officer supplemented importantly the statistical data supplied during progress of work by Major Yerkes. As the results of psychological examining indicated clearly the value of the work for the elimination of men mentally defective, the balancing of organi-

sations in mental strength, the classification of men for the assistance of personnel officers in the camps, and the indication of men of exceptional intelligence to be charged with special responsibility or sent to Officers' Training Camps, and as all reports of the inspector were favorable, the Surgeon General in December 1917 recommended to the Secretary of War the continuance and the extension of psychological examining to the entire army. This recommendation, after careful inquiry concerning the value of the work by the Training Committee of the War College Division of the General Staff, was approved by the Secretary of War who directed the Surgeon General to prepare a plan for the proper conduct of the proposed work.

A comprehensive plan was promptly prepared by the Staff of the Section of Psychology, Division of Neurology, Psychiatry and Psychology. This plan provided for suitably trained personnel, special psychology building in each camp, and necessary apparatus and printed materials. It was fully approved by the Secretary of War, Jan. 19, 1918, and the Surgeon General was authorized to create a Division of Psychology in his office, and to put the plan of psychological examining into effect.

A school for military psychology was immediately organized in connection with the Medical Officers' Training Camp, Fort Oglethorpe, Georgia, in which a maximum of 100 student officers and approximately 100 psychologists enlisted in the Medical Corps could be trained simultaneously during a period of two months. This school was opened February 4 with Captain William S. Foster, San. C., N.A. as senior instructor. Between this date and July 1, 1918, approximately 70 officers of the Sanitary Corps and 250 men enlisted in the Medical Corps were given special training in military psychology.

Plans for suitable psychology buildings were prepared and duly submitted to the Quartermaster General. Subsequently the Equipment Committee of the General Staff disapproved the expenditure for these buildings pending Congressional appropriation for the purpose, and by special request of the Secretary of War, existing buildings were assigned for psychological examining in most of the divisional training camps.

The necessary apparatus and printed materials for the examining of 500,000 soldiers were speedily prepared, and during April and May, psychological examining staffs were organized in twenty-five National Army and National Guard camps. During the month of June additional apparatus and printed materials were manufactured for the examining of 1,000,000 soldiers.

On July 1, 1918, psychological examining was in progress in twenty-eight army camps and in three General Hospitals. Seventy-nine officers of the Sanitary Corps were on duty in these stations, and in the Division of Psychology, Surgeon General's Office. Approximately 100 non-commissioned officers and privates of the Medical Corps especially trained in military psychology had been assigned to duty in examining stations, and somewhat more than 100 were in training at the school of military psychology,

Fort Oglethorpe. Up to April 27, 1918, when new methods were put into use, approximately 140,000 soldiers had been examined. On July 1, 1918, 600,000 had been examined. Of this number, slightly more than 0.25% had been recommended by psychological examiners to psychiatrists for discharge because of mental deficiency, and about 0.5% had been recommended for assignment to service organizations for development battalions because of mental inferiority.

Psychological examinations which were originally conducted in wards of Base Hospitals, are now made in a psychology building located usually in the Depot Brigade. In this building, drafted men are examined promptly on reporting to camp. Their intelligence ratings are immediately transmitted to the Personnel Officer of the camp. All cases of mental deficiency, or those for which psychiatric examination is indicated as desirable, are referred to the psychiatrist.

The aim of the Division of Psychology, Surgeon General's Office, is to develop a psychological center in each military training camp to which all psychological problems of military assignment, training, discipline, morale, and intelligence may be referred by officers of the line or staff. Such a center exists in twenty-five camps, and the service is being extended as rapidly as available personnel permits.

(Publication approved by the Board of Publications, Office of the Surgeon General.)

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## *THERMO-ELECTRIC ACTION WITH THERMAL EFFUSION IN METALS: A CORRECTION*

By Edwin H. Hall

Jefferson Physical Laboratory, Harvard University

Communicated August 3, 1918

The last paragraph of my paper in these Proceedings for April, 1918, dealt incorrecly with the effect of 'hypothesis (B)' the hypothesis that free electrons in the interatomic spaces of an unequally heated metal bar have a tendency like that which in ordinary gases produces the phenomena of thermal effusion.

In a common gas the condition of equilibrium maintained by thermal effusion is not $p$ constant but $p \propto T^{\frac{1}{2}}$, where $T$ is the absolute temperature. In dealing with the free electrons, for which the $R$ of the equation $pv = NRT$ may not be a constant, we have as the condition which thermal effusion tends to create

$$p \propto (RT)^{\frac{1}{2}} \tag{1}$$

whence

$$dp = \frac{p}{2}\left(\frac{dR}{R} + \frac{dT}{T}\right). \tag{2}$$

If we take one gram of electrons, each of mass $m$, we have

$$pv = \frac{1}{m} RT, \tag{3}$$

and by substitution in (2) we get

$$dp, \text{ or } (dp)_s = \frac{1}{2v} \cdot d\ (pv). \tag{4}$$

This is the difference of pressure which the free electrons within a metal could bear without drift and without the help of electrical force. It may be called the *static difference of pressure*, $(dp)_s$.

If, now, there exists within a metal a free-electron pressure $dp > (dp)_s$, the excess

$$dp - (dp)_s, \text{ or } dp - \frac{1}{2v} \cdot d\ (pv), \tag{5}$$

will be the true mechanical driving force operating to maintain a drift of the electrons. Hence the introduction of 'hypothesis (B)' requires the substitution of $dp - (1 \div 2v)\ d(pv)$ for $dp$ in the second member of equation (4), of my paper referred to above, and makes the whole equation read

$$(P_c - P_h) + \int^c \frac{k_f}{k} \cdot dP_f + \int_h^c \frac{k_a}{k} \cdot dP_a = \frac{1}{Ge} \int_c^h \frac{k_f}{k} \left( vdp - \tfrac{1}{2} d\ (pv) \right), \tag{6}$$

where $k$ has the same meaning as the $(k_a + k_f)$ used in the original equation.

This is the expression for the 'virtual e.m.f.' resident in a detached metal bar having one end at $T_h$ and the other at $T_c$. In my previous discussion of this matter, not written out in my April paper, I had proceeded as if the factor $(k_f \div k)$ could be put with the $(pv)$ so as to give $d\,[(k_f \div k)(pv)]$. This error led to my giving figure 4 of that paper as the graphical representation, on the $P$-$V$ plane, of the virtual e.m.f. in question.

The second member of equation (6) can be put into the form

$$\frac{1}{Ge} \int_c^h \frac{k_f}{k} \left( \tfrac{1}{2} vdp - \tfrac{1}{2} pdv \right),$$

and the graphical representation of this expression is given in figure 1. Here $A\ D$ is the $pv$ line for one gram of free electrons in the metal; $A'\ D'$ is obtained from $A\ D$ by applying the proper value of the factor $(k_f \div k)$ to each value of $v$; and $A'\ D'$ is obtained for $A\ D$ by applying the proper value of the factor $(k_f \div k)$ to each value of $p$. $A''\ D''$ is obtained from $A'\ D'$ by taking one-half of each value of $v$, so that the area $E\ A''\ D''\ G$ represents the part $\frac{1}{2} \int_c^h \frac{k_f}{k} \cdot vdp$ of (7). $A_1''\ D_1''$ is obtained from $A_1'\ D_1'$ by taking one-half of each value of $p$, so that $A_1''\ D_1''\ d\ a$ represents the part $-\frac{1}{2} \int_c^h \frac{k_f}{k} \cdot pdv$ of (7).

The sum of these two areas, multiplied by $(1 \div Ge)$, represents the virtual e.m.f. in question.

If the line $A\,D$ were of constant $pv$, as it might be if we were dealing with an isothermal 'alloy bridge,' the two shaded areas would be equal and their sum would equal $E\,A'\,D'\,G$. That is, there would be no tendency to 'thermal effusion' in such a case, and the virtual e.m.f. would reduce to

$$\frac{1}{Ge}\int_c^h \frac{k_b}{k}\cdot v\,dp,$$

the value it has under hypothesis (A).

The total effective e.m.f. for a closed circuit made up of two different metals and two isothermal alloy bridges is represented by $(1 \div Ge)$ times the sum of two areas like $A''\,B''\,C''\,D''$ and $A_1'\,B_1'\,C_1'\,D_1'$ in figure 2, where the path $A\,B\,C\,D$ represents the $pv$ changes of one gm. of *free* electrons throughout the circuit.

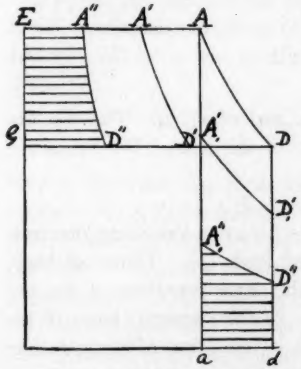


FIG. 1

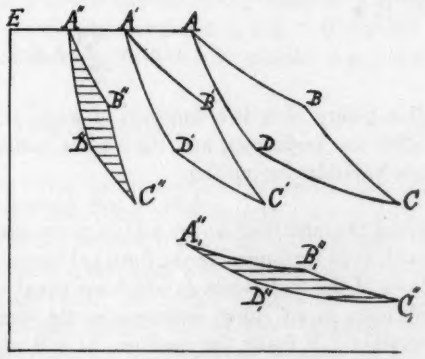


FIG. 2

The total effective e.m.f. of the circuit is not, as I have previously believed it to be, necessarily the same under hypothesis (B) as under hypothesis (A). For the latter case it is represented in figure 2 by $(1 \div Ge)$ times the area $A'\,B'\,C'\,D'$, the boundary of which corresponds to the changes of pressure and volume undergone by 1 gram of electrons, in part free electrons and in part associated electrons, in going around the circuit as part of a current.

I had overlooked the interesting fact that, when we have to do with thermal effusion, which overrides the tendency to equality of pressure throughout a gaseous body, we can no longer calculate the work done by or on a body of gas by means of a mere diagram of its pressure-volume changes. Thus, in a vertical cylinder containing a column of air, kept warmer at the top than at the bottom, a porous partition extending partly across the cylinder wou d constantly transmit air upward through its interstices, to be returned downward past the edge of the partition, and work could thus be done by means of air acting in a $pv$ cycle of no area.

# INVARIANTS AND CANONICAL FORMS

By E. J. Wilczynski

Department of Mathematics, University of Chicago

Communicated by E. H. Moore, August 7, 1918

Every student of the theory of invariants has observed the fact that the coefficients of a unique canonical form are invariants. But a general *a priori* proof of this fact, sufficiently general to cover all of the cases needed in the applications, seems to be lacking. It is the purpose of this paper to furnish such a proof, making use only of the abstract principles which are common to all known invariant theories.

We begin by giving a brief outline of some of these invariant theories, so that we may have these instances in mind when we formulate our general theory. Consider first a binary $n$-ic,

$$(p) = \sum_{i=0}^{n} p_i x_1^i x_2^{n-i}. \tag{A}$$

This binary *form* is a function of $p_0, p_1, \ldots, p_n$ and of $x_1, x_2$. The $p$'s are called the *coefficients*, and the $x$'s the *variables* of the form. We introduce new variables by putting

$$\bar{x}_i = \alpha_{i1} x_1 + \alpha_{i2} x_2 \quad ,(i = 1, 2),$$

where the quantities $\alpha_{ik}$ are arbitrary constants with a non-vanishing determinant, thus transforming the form $(p)$ into a new form $(\bar{p})$. Those combinations of the coefficients $p_k$ which are equal to the same functions of the coefficients $\bar{p}_k$ are called *invariants* of the form. In the classical theory of invariants it is really the *equation* $(p) = 0$ which is the object of study rather than the *form* or *function* $(p)$. Consequently the additional transformations, operating upon the coefficients only, $\bar{p}_i = \lambda p_i$, $(i = 0, 1, 2, \ldots, n)$, where $\lambda$ is an arbitrary constant, are introduced. An invariant of the *equation* must remain unaltered by these transformations also.

A second invariant theory is concerned with the class of linear differential expressions or forms

$$p_0 \frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + \ldots + p_n y, \tag{B}$$

where $p_0, p_1 \ldots, p_n$, and $y$ are functions of $x$. The *coefficients* $p_k$ in this case are functions of $x$, whereas in instance (A) they are numbers, real or complex. In instance (A) the variables $x_1, x_2$ are also numbers; in (B) the variables $y, dy/dx, \ldots, d^n y/dx^n$ are functions. In this case we may even think of $y$ as the *only* variable, since the variables $dy/dx$, etc., are determined when $y$ is given.

Let us now transform the variable $y$ by putting $y = \lambda(x)\bar{y}$, where $\lambda(x)$ is an arbitrary function. Then (B) goes over into a new differential form $(\bar{B})$

whose coefficients $\bar{p}_k$ depend upon $p_0, p_1, \ldots, p_n$ and $\lambda$. The corresponding invariants are of considerable importance. Here also we must distinguish between the theory of invariants of forms, and the theory of invariants of equations.

As a third instance, let us consider the class of analytic functions

$$f(x) = p_0 + p_1 x + p_2 x^2 + \ldots + p_n x^n + \ldots, \qquad (C)$$

where the coefficients $p_0, p_1 \ldots, p_n, \ldots$, and the variable $x$ are complex numbers. We introduce transformations of the form

$$\bar{x} = \frac{\alpha x + \beta}{\gamma x + \delta}, \quad \alpha\delta - \beta\gamma \neq 0,$$

where $\alpha, \beta, \gamma, \delta$ are arbitrary constants, and attempt to find invariants of $f(x)$ under all such transformations. In this case we are dealing with a form which has infinitely many coefficients, forming a denumerable set.

We shall list one further instance of our general theory. Let $K(x, \xi)$ be a real continuous function of $x$ and $\xi$ in the region $0 \leqq x \leqq 1$, $0 \leqq \xi \leqq 1$, and let $\varphi(\xi)$ be a real continuous function of $\xi$ in the interval $0 \leqq \xi \leqq 1$. Then

$$I(x) = \int_0^1 K(x, \xi)\, \varphi(\xi)\, d\xi \qquad (D)$$

may be regarded as a form whose value, as function of $x$, depends upon the choice of the $K$ and $\varphi$ functions. We may think of the functions of $x$ to which $K(x, \xi)$ reduces, for all special values of $\xi$ between 0 and 1, as the coefficients of the form. Thus the number of these coefficients is non-denumerable, even if the function $K(x, \xi)$ be regarded as given. As to the variables of the form $I(x)$ we still have two choices. We may think of the *function* $\varphi(\xi)$ as ranging over the class of all continuous functions and regard $\varphi(\xi)$ as the only variable of the form; or else we may consider the *functional values* of $\varphi(\xi)$ as the variables. In the latter case, the *range* of the variables would be the class of real numbers, and the number of variables would be continuously infinite.

We may transform (D) by putting

$$\varphi(\xi) = \lambda(\xi)\, \bar{\varphi}(\xi),$$

where $\lambda(\xi)$ is an arbitrary continuous function of $\xi$. This will transform $I(x)$ into $\bar{I}(x)$, where

$$\bar{I}(x) = \int_0^1 \bar{K}(x, \xi)\, \bar{\varphi}(\xi)\, d\xi, \quad \bar{K}(x, \xi) = K(x, \xi)\, \lambda(\xi).$$

In particular it will be possible to choose $\lambda(\xi)$ in such a way as to make

$$K(\tfrac{1}{2}, \xi) = 1$$

except for those values of $\xi$ for which $K(\tfrac{1}{2}, \xi) = 0$; the resulting integral may be said to be in its *canonical form*.

For the purposes of our general theory, which shall include all of the instances mentioned as well as infinitely many others, *we postulate a class* $[F]$ *of forms, or functions of two general arguments;* $F(p, x)$. As the notation indicates, the arguments of such a form are of two kinds, the *coefficients* $p$, and the *variables* $x$. Both $p$ and $x$ are supposed to be *general variables* in the sense of E. H. Moore, each argument varying on its own range. This range may be a continuous or discrete class of numbers, or a class of functions, or any other well-defined range; it may be the same for $p$ and $x$, or different Finally the range over which $F$ varies may be different from either or both of the ranges of $p$ or $x$. In the language of ordinary analysis this general formulation includes single forms or systems of forms, whose coefficients and variables may be finite or infinite in number, and in the latter case denumerably or non-denumerably infinite.

We further assume that *the class of forms* $[F]$ *is well-defined.* This means that a criterion is given by means of which we may decide whether a given form does or does not belong to $[F]$. This criterion will also enable us to distinguish between the coefficients and the variables of the form

*Two forms of the class* $[F]$ *are said to be identical if their corresponding coefficients are equal.* Equality of the correspondng variables is *not* required for the identity of two forms. For this reason it is frequently convenient to suppress the notation for the variables in the symbol for a form, and to use the simpler symbol $F = (p)$ to replace $F(p, x)$.

*We postulate, in the second place, a group of transformations, which operates upon the variables of a form* $F$ *of* $[F]$ *and transforms every* $F$ *of* $[F]$ *into another form* $\bar{F}$ *of the same class. The coefficients* $\bar{p}$ *of* $\bar{F}$ *will depend on the coefficients* $p$ *of* $F$ *and upon elements which occur in the transformation used. We assume that we obtain in this way a new group* $G$ *operating upon the coefficients of the form.* $G$ *is said to be induced by* $g$.

In this postulate the word group is uded in the usual sense. Thus we call a set of operations a group if the identity is included among its operations, if the operations possess an associative law of combination, if the product of any two operations of the set and also the inverse of every operation of the set belong to the set. The two groups, $g$ and $G$, may however contain a finite number of operations or infinitely many; in the latter case they may be discrete or continuous; if they are continuous, they may be finite or infinite, in the sense of Lie.

*By an appropriate modification of the group* $G$, *we may include in our theory not only the invariants of forms, but also the theory of invariants of equations and systems of equations.* This has been pointed out already in connection with our preliminary discussion of instance (A).

*If there exists at least one transformation in the group* $G$ *which, when applied to a form* $F_1$ *of* $[F]$, *transforms* $F_1$ *into a form* $F_2$, *the two forms* $F_1$ *ad* $F_2$ *are said to be equivalent under the group* $G$.

Let us now define a proper sub-class $[\Phi]$, of $[F]$, by imposing some property upon the coefficients $p$ which is not satisfied by all forms of $[F]$. It is assumed that this property is well defined, so that we may be able to decide whether a given form $F$, of $[F]$, does or does not belong to the sub-class $[\Phi]$. Let us assume that, for every form $F$ of $[F]$, there exists in $G$ at least one transformation $S$ which transforms $F$ into an equivalent form of the sub-class $[\Phi]$. *We shall then say that* $\Phi = S[F]$ *is a canonical form of* $F$. It may happen that such a canonical form equivalent to $F$ under the group $G$, and belonging to the sub-class $[\Phi]$ exists merely for those forms of $F$ which do not belong to a well-defined sub-class $[\Psi]$ of $[F]$. We shall speak of the forms of the sub-class $[\Psi]$ as *exceptional* forms, and call all the other forms of $[F]$ *generic* forms. Of course the term, *general* form of $[F]$, includes both the exceptional and the generic forms. Under these circumstances we shall still speak of $\Phi$ as a canonical form of $F$, but we shall add the qualifying phrase *for the generic case* whenever the distinction becomes necessary.

In general there will be many transformations of $G$ which transform every generic $F$ into a canonical form of the sub-class $[\Phi]$. If all of these transformations transform every generic $F$ into the same form $\Phi$, of the sub-class $[\Phi]$, we shall say that *the canonical form* $\Phi$ *is unique.*

It remains to define the term *invariant.* A function $I$, of the coefficients $p$ of a form $F$, is an absolute invariant under the broup $G$, if it is equal to the same function of the corresponding coefficients of any form $\overline{F}$ which is equivalent to $F$, by means of a transformation of the group $G$.

This notion may be regarded as including also the notion *covariant.* For, we may replace the given form $F$ by another form, or system of forms, who coefficients depend also upon the *variables* of $F$.

We are now ready to prove the following theorem.

*Let* $[F]$ *be a well-defined class of forms, and let* $[\Phi]$ *be a proper sub-class of* $[F]$. *Let* $G$ *be a group of transformations which transforms every form of* $[F]$ *into a form of the same class. By a generic form of* $[F]$ *we mean one which is equivalent to a form of* $[\Phi]$ *under* $G$. *Then, there exists, for every generic form of* $[F]$ *at least one transformation in* $G$ *which transforms* $F$ *into a canonical form of the sub-class* $[\Phi]$. *If this canonical form is unique, its coefficients are one-valued absolute invariants of the form* $F$ *for the group* $G$.

*Proof.*—Let $F$ be a generic form of $[F]$, and let $\Phi$ be its canonical form. Let $S$ be the most general operation of $G$ which transforms $F$ into $\Phi$. We shall have symbolically

$$S(F) = \Phi. \tag{1}$$

The operation $S$ will depend, in general, upon the coefficients $p$, of $F$, and may contain, besides, arbitrary elements in great number. The canonical form $\Phi$ belongs to the sub-class $[\Phi]$ of $[F]$. Since this canonical form is, by hypothesis, unique, its coefficients $\pi$ are independent of the arbitrary elements which may occur in $S$, and they are one-valued functions of the coefficients

of $F$ in the sense, that when $F$ is given, the coefficients of $\Phi$ are determined uniquely. We may express this symbolically by the equation

$$\pi = U(p), \tag{2}$$

where $U(p)$ is the symbol for a one-valued function of the coefficients $p$.

Now let $T$ be any transformation of $G$, and let

$$\overline{F} = T(\overline{F})$$

be any form of $[F]$ equivalent to $F$ under $G$. Since we assumed that $F$ was generic, and since

$$F = T^{-1}(F),$$

we conclude that $\overline{F}$ is also generic. In fact we find

$$S(F) = ST^{-1}(F) = \Phi,$$

so that $ST^{-1}$ is an operation of $G$ which transforms $\overline{F}$ into a canonical form of the sub-class $[\Phi]$.

Let $\overline{S}$ be the most general operation of $G$ which transforms $\overline{F}$ into a canonical form of the sub-class $[\Phi]$, so that

$$\overline{S}(\overline{F}) = \overline{\Phi},$$

and let $\overline{\pi}$ denote the coefficients of $\overline{\Phi}$. These coefficients will depend on the coefficients $\overline{p}$ of $\overline{F}$, in exactly the same way as the coefficients $\pi$ of $\Phi$ depend upon the coefficients $p$ of $F$. That is, we shall have, in a manner analogous to (2),

$$\overline{\pi} = U(\overline{p}). \tag{3}$$

We also know that

$$\overline{\Phi} = \Phi;$$

for we have seen already that $ST^{-1}$ will transform $\overline{F}$ into $\Phi$, and we are assuming that every generic form $\overline{F}$ of $[F]$ has a ***unique*** canonical form of the sub-class $[\Phi]$.

But, if two forms of a class are equal, their corresponding coefficients are equal. Therefore we have $\overline{\pi} = \pi$ or, according to (2) and (3).

$$U(\overline{p}) = U(p).$$

In other words the coefficients of $\Phi$ are indeed absolute invariants of $F$ under the group $G$, and therefore our theorem is demonstrated.

If the forms of the class $[F]$, which are generic forms from the point of view of the canonical form chosen, do not constitute the whole of $[F]$ there will remain in $[F]$ certain exceptional forms constituting a sub-class $[H]$ of $[F]$. But the theorem may be applied to these exceptional forms as well, whenever a unique canonical form exists for a generic one of the exceptional forms; but of course, the canonical form in this case will not belong to the class $[\Phi]$, but

to a certain sub-class $[X]$, of $[H]$. If the forms of $[H]$ are not all generic from this new point of view, we may adopt a new canonical form for the exceptional ones, and continue in this way.

We have observed that the coefficients of a unique canonical form are absolute invariants, and moreover one-valued invariants, in the sense, that their values are uniquely determined as soon as the coefficients of the form $F$ are given. In the ordinary theory of algebraic invariants, it is at once apparent that these invariants are algebraic functions of the coefficients of $F$. Consequently it follows from their one-valuedness that, in this case, these invariants are *rational* functions of the coefficients. In the theory of invariants of linear differential equations, the uniquenes of a canonical form gives rise to invariants which are rational functions of the coefficients of the differential equation and of their derivatives.

There are many cases in which a $k$-valued canonical form is obtained rather than a unique one. That is, if we resume our terminology, the sub-class $[\Phi]$ contains not merely one, but exactly $k$ forms $\Phi_1, \Phi_2, \ldots, \Phi_k$, each of which is equivalent to $F$ by a transformation of the group $G$. *The coefficients of these canonical forms will still be absolute invariants of $F$, but they will be $k$-valued functions of its coefficients.* It is obviously possible to find an equation of degree $k$ with one-valued invariants as coefficients, of which these $k$-valued invariants are the roots. In the theory of algebraic invariants we obtain in this way irrational invariants, as roots of an equation whose coefficients are rational invariants.

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## TYPES OF PHOSPHORESCENCE

By Edward L. Nichols and H. L. Howes

Department of Physics, Cornell University

Communicated, August 1, 1918

The existence of phosphorescence of exceedingly short duration was long ago revealed by the phosphoroscope of Becquerel but, until very recently the afterglow of luminescent bodies has been studied quantitatively, only where it is of comparatively long duration. Curves of decay were supposed to be all of the same character. It was assumed that the law of diminution of brightness, as expressed by the equation

$$I = \frac{1}{(a_1 + b_1 t)^2} + \frac{1}{(a_2 + b_2 t)^2}$$

was of general application and that the phosphorescence of various substances differed only in color, brightness and duration.

The measurements of Waggoner[1] and of Zeller[2] on phosphorescence of short duration tended to confirm this view. On the other hand the observations of

Ives and Luckiesh[3] on the effect of temperature upon the curves of decay of certain phosphorescent sulphides might seem to demand a modification of the usual law in certain cases, while the studies of the phosphorescence of gases by C. C. Trowbridge[4] and of paraffine at liquid air temperatures by Kennard[5] indicate that for the range covered by their measurements the law of decay cannot be expressed by a summation of $1/(a + bt)^2$ terms.

In a recent investigation[6] we have found the exceedingly brief phosphorescence of the uranyl salts, which, although very brilliant lasts only for about 0.03 seconds, to be of an entirely different type. It decays very slowly at first and later very rapidly, whereas in all cases previously studied the opposite is true. Measurements just completed on the luminescence of calcite indicate that this newly determined form of decay curve is not confined to the uranyl salts.

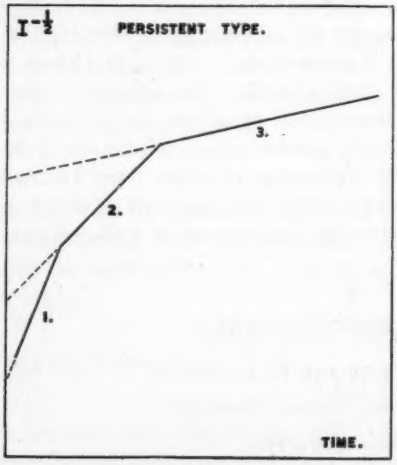


FIG. 1

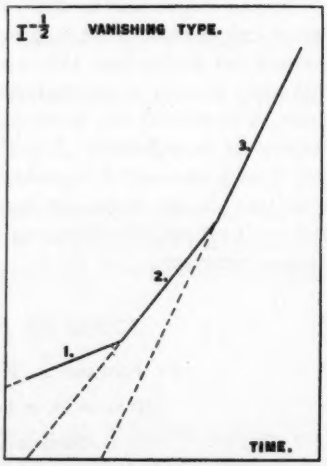


FIG. 2

We propose therefore to recognize two distinct types of phosphorescence and to designate them as *persistent phosphorescence* and *vanishing phosphorescence.* They are sharply distinguished from one another by the following characteristics.

*Types of phosphorescence.—1:* (Characteristic of persistent phosphorescence.) This type has a curve of decay made up of a succession of linear process, of diminishing slope as we proceed from the origin of time (fig. 1). Three or more such processes have been found in all cases which have been studied through a sufficient range.

*2:* (Characteristic of vanishing phosphorescence.) This type has a curve of decay made up of a succession of linear processes, of increasing slope as we proceed from the origin of time (fig. 2).

It is understood that in these specifications and elsewhere, unless otherwise stated, the curve of decay is plotted with time from the close of excitation as abscissae and the reciprocal of the square root of intensity ($I^{-\frac{1}{2}}$) as ordinates. A linear process is any straight portion of the graph; as *1*, *2* or *3* in figures 1 and 2. The inference is that the more or less abrupt changes of slope are related to and indicative of real changes in the *processes* by which the emission of light from the phosphorescing body is being maintained.

A distinguishing criterion of type is found in the sign of the intercepts of the various processes at the origin of time. In type 1 the intercepts are all positive; in type 2 the intercepts of process 2 and 3 are negative.

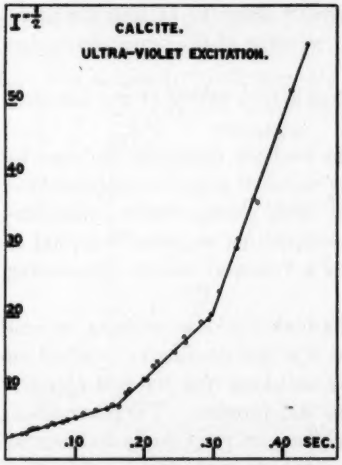


FIG. 3

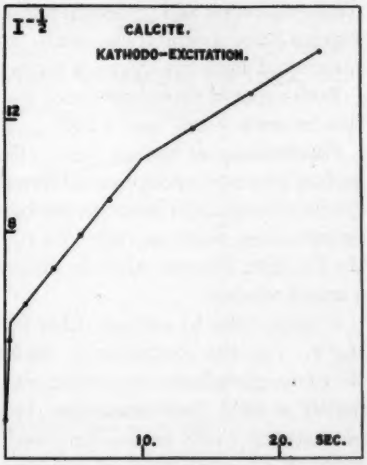


FIG. 4

The passage from one process to the next is presumably never actually discontinuous, as in the above diagrams, but it is sometimes very abrupt as in the curves of figures 3 and 4 which are from our recent measurements of the phosphorescence of calcite. Sometimes the transition from slope to slope is very gradual; so that the first and second processes tend to merge into a curved line. We have shown,[6] in the case of the uranyl salts that the location of the knees depends on the intensity of the excitation and that distributed knees —so to speak—occur when the exciting light penetrates the crystal and being absorbed produces luminescence of decreasing intensity in successive layers; also that where the conditions are such that excitation is confined chiefly to the surface, the knees are well defined.

Similar modifications of the curve occur in kathodo-excitation, and distributed knees are then probably due to a kathodic discharge containing particles of varying velocity.

*Both types may occur in a single substance.*—Shortly after the completion of our experiments on the vanishing phosphorescence of the uranyl compounds, Misses Wick and McDowell[7] found that some of the same salts, notably $K.UO_2(NO_3)_2$ and $K_2UO_2(NO_3)_4$ when exposed to the kathode discharge at the temperature of liquid air glowed for many seconds after the close of excitation with decay curves of the persistent type. At $+20°$ the effect is either absent or excessively feeble. Not all the uranyl salts, even among those that are reasonably stable in vacuo, respond to the action of the kathode discharge to a measureable extent.

The Franklin Furnace calcites, as we discovered in our recent investigation,[8] have the same remarkable property, i.e., vanishing phosphorescence under photo-excitation and persistent phosphorescence under the kathode discharge. Figures 3 and 4 exhibit the vastly different behavior of the same calcite after the close of these two modes of excitation.

Both types of phosphorescence are obtainable with calcite at any temperature between $-180°$ and $+300°$.

*Independence of the two types.*—Since the kathode discharge modifies the surface layers of substances subjected to its action, it might be supposed that calcite crystals after kathodo-bombardment would show persistent phosphorescence when photo-excited. To test this supposition we placed a crystal of the Franklin Furnace calcite in the bottom of a V shaped vacuum tube having a quartz window.

It could thus be excited either by the kathode discharge or from an iron spark. Previous excitation by the kathode rays had no observable effect on the photo-phosphorescence which was of the vanishing type and had approximately at least the normal color, brightness and duration. The photo-phosphorescence could be superimposed upon the persistent kathodo-luminescence, as a fleeting effect, at any time during the life of the latter either after or before the close of the kathodo-excitation. Apparently the two were entirely independent of each other.

Misses Wick and McDowell had previously made a similar observation in their study of the kathodo-phosphorescence of the uranyl salts; i.e. that the kathode rays do not render them capable of persistent photo-phosphorescence.

*Apparent occurrence of both types with a single source of excitation.*—Willemite is one of the most brlliant of luminescent substances. The afterglow is commonly fleeting, but masses are occasionally found which exhibit phosphorescence of long duration. Such specimens are persistent under photo-excitation and kathodo-excitation alike. The phosphorescence of the other variety, unlike that of calcite and of the uranyl salts, is of short duration under both kinds of excitation.

The determination of the decay of phosphorescence of a specimen of the latter variety, using the disk phosphoroscope, gave curves like that plotted in figure 5. Processes 1 and 2 are of type 1 (vanishing) but these are followed by a process 3 of lesser slope.

This is obviously a composite curve due to the superposition of phosphorescence of both types. It does not follow however that it is a true example of a *single substance* brought to both types of phosphorescence by photo-excitation. The evidence is to the contrary.

1. The two varieties of willemite are commonly associated. One of our specimens contains parallel veins of the persistent form and of willemite of short duration in the same matrix.

2. Willemite is associated with the Franklin Furnace calcite.

That the vanishing phosphorescence in the curve in figure 5 may be due to an admixture of calcite is suggested by the following observations.

*a.* When the disk of the phosphoroscope, coated with the powdered willemite and exposed to the light of the iron spark, was driven rapidly, the region nearest the spark in the direction of revolution but shielded from the direct light was a brilliant yellow-green, the regions approaching the spark but

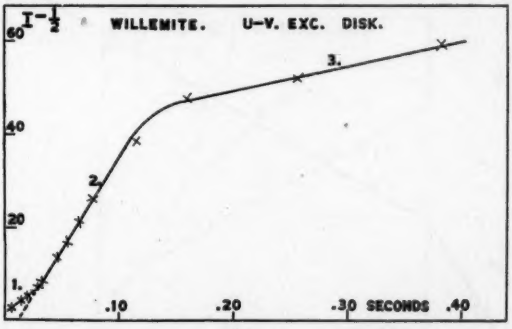


FIG. 5

shielded, were of a dark *blue*-green. This is the effect which would be expected were calcite present, since its red-yellow phosphorescence would give the yellowish caste during the first process of decay but would vanish before the revolution of the disk was completed.

*b.* When the collimator of a spectroscope was directed to the rapidly revolving disk the spectrum of the phosphorescent willemite appeared as a broad continuous band extending from the extreme red to the blue. The crest was approximately at 0.54 $\mu$ and there was suspicion of a weak region in the yellow.

At a somewhat lower speed, which allowed a longer interval of time between excitation and observation the red end of the spectrum disappeared and at a much lower speed the yellow and brightest part of the green vanished leaving a much narrower band in what had been a comparatively feeble region, with its crest in the blue green at about 0.52 $\mu$.

The former brilliant crest at 0.54 $\mu$ was now very dim and lay near the edge of the persistent band. This persistent crest coincides in position with that of the fluorescence spectrum of a willemite of long duration determined spectrophotometrically some years ago.[9]

Obviously we have in this spectrum two, and perhaps three overlapping bands. A very rapid red band, presumably due to calcite, the band of the willemite of short duration and a persistent band of small intensity indicative of the presence of willemite of long duration.

Not all specimens of willemite have the complicated curve of decay above described. Waggonèr,[1] (1908) in his studies of phosphorescence of short duration, determined the beginnings of the curves of decay of two willemites; one of the rapid and one of the persistent variety. He used the Merritt phosphoroscope having a range up to 0.06 seconds. His curves are of the persistent type.

*Not all phosphorescence of quick decay is of the vanishing type.*—That the type of phosphorescence is not altogether a question of duration we have abundant

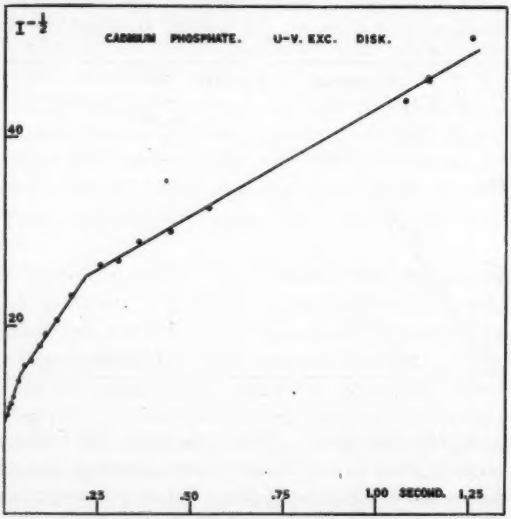


FIG. 6

evidence. The willemite of quick decay measured by Waggoner is of much shorter measurable duration than the Franklin Furnace calcites, yet its curve of decay is clearly of the opposite type. The same is true of various phosphorescent compounds prepared by Waggoner, i.e., $ZnCl_2$, CdCl, $CdSO_4$,—each with a trace of $MnSO_4$ added, which were heated to redness with a flux of $Na_2SO_4$. Although their phosphorescence is of brief duration they gave decay curves of the persistent type.

There is this distinction between the luminescence of such substance and vanishing phosphorescence. The latter becomes completely extinct almost immediately after it ceases to be of measurable intensity; as indicated by the steepness of the second and third processes (fig. 2). In the persistent type

of short duration the loss of light during the initial process may go to the very limit of visibility within a few hundredths of a second or less, but if the threshold is not actually crossed the phosphorescence may remain, barely visible in a completely dark room, for a considerable time. Waggoner remarks (on page 216 of his paper) "*It will be noted that in practically all the substances studied, the measurable portion of the decay is over in 0.07 seconds. Some of the substances may, however, be seen in a dark room for a very much longer time.*"

Phosphorescence of short duration, when *not* of the vanishing type, may be considered simply as a case of the persistent type in which the rapid initial process reduces the intensity nearly to or beyond the threshold of visibility. The subsequent processes, though gradual are therefore too faint for observation or invisible. The brightness of the first process may be as great or greater in cases of quick decay than where the phosphorescence continues of measurable intensity for a long time. Zeller[2] who has studied the beginnings of decay of the phosphorescence of various persistent compounds has made note

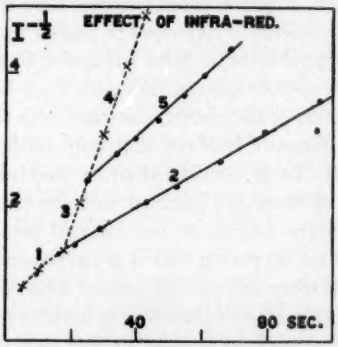


FIG. 7

of the fact that duration is not in any simple way related to initial brightness. In a group of phosphorescent cadmium compounds, in particular, some that were too dim for measurement with the phosphoroscope remained visible for a long time while those of greatest initial intensity were of very short duration.

An interesting substance which came to our notice during these investigations through the kindness of Mr. W. L. Lemcke is a cadmium phosphate prepared by Mr. W. S. Andrews. Under the iron spark it is excited to a fine white phosphorescence having a measurable duration of about one second. We determined its curve of decay with the disk phosphoroscope. As may be seen from figure 6 it is of the persistent type and so far as the white afterglow is concerned might be classed as of rather short duration; but a very faint ruddy phosphorescence remains for a much longer time. This specimen, as viewed on the rotating disk is remarkable for the succession of color effects which it exhibits. Pink, very fleeting, is followed by nearly pure white, then for an instant by blue which goes over into a very persistent pink, turning ruddy as decay progresses.

Such color changes in other substances have been shown to be due to the existence in the phosphorescence spectrum of overlapping bands having different rates of decay and the striking display would seem to indicate an unusual complexity of composition. We were surprised, therefore, to learn from Mr. Andrews that for the production of this brilliant white phosphorescence cadmium phosphate of exceptional purity is necessary, and that there is no admixture of other metallic salts, as in the preparation of the phosphorescent sulphides.

Curves exhibiting the new vanishing type of decay, above described, resemble nothing in the earlier work on phosphorescence so much as the curves obtained as a result of the action of infra-red light on Sidot Blende.[1] Figure 7 exhibits this phenomenon. Processes 1 and 2 together show the pure decay curve, without infra-red, but if the infra-red is applied after 18 seconds of decay the more rapid process 3–4 ensues. If, however, the infra-red is removed at the end of 25 seconds the decay resumes a slower rate, as shown by process 5. Processes 1, 3 and 4 taken together show a striking resemblance to the vanishing type of decay. Attempts to hasten the decay of calcite or of the phosphorescent uranyl salts by infra-red action however have yielded no result. Here the successively steeper processes must be due to a change in the rate of recombination of the ejected electrons with the active phosphorescing groups. It is commonly believed that only certain groups of particles are active, the majority being considered to be inactive. It is evident that these changes in rate of decay are inherent with the crystal and must be due to a more or less sudden change in the internal conditions. The electric fields within a crystal are so strong that it is perhaps not surprising that the infra-red field, applied from the outside, cannot affect either the phosphorescent action in the active groups or increase the number of free electrons. The sudden change in the rate of decay may be due to a change in the electric field concomitant with the changed orientation of the charged particles, which drives the ejected electrons in greater proportion to the non-active parts of the molecule.

[1] Waggoner, *Physic. Rev., Ithaca*, (Ser. 1), **27**, 1908, (209).
[2] Zeller, *Ibid.*, **31**, 1910, (367).
[3] Ives and Luckiesh, *Astroph. J., Chicago*, **36**, 1912, (330).
[4] Trowbridge, C. C., *Physic. Rev.*, **32**, 1911, (129).
[5] Kennard, *Ibid.*, (Ser. 2), **4**, 1914, (278).
[6] Nichols and Howes, *Ibid.*, **9**, 1917, (292).
[7] Wick and McDowell, *Ibid.*, **11**, 1918, (421).
[8] To be described in a forthcoming number of the *Physical Review*.
[9] Nichols and Merritt, *Ibid.*, (Ser. 1), **28**, 1909, (349).
[10] Nichols and Merritt, *Studies in Luminescence, Carnegie Inst., Washington, Pub.*, **152**, 1912.

# THE SMITHSONIAN 'SOLAR CONSTANT' EXPEDITION TO CALAMA, CHILE

By C. G. Abbot

Smithsonian Astrophysical Observatory

Communicated August 16, 1918

As early as 1903 the observations of the Smithsonian Astrophysical Observatory suggested the view that the solar radiation varies over a range of several per cent within intervals as short as a few days or weeks. We were measuring the radiation of the sun at the earth's surface. The measurements comprised determinations of the heating effect of the solar rays on the blackened surface of the pyrheliometer, which measures all rays of the spectrum as found in 'white light,' and also the heating effects at all parts of the solar spectrum as detected by the bolometer, including ultra-violet, visible and infra-red rays. We measured at intervals on every clear day as the sun declined from high altitudes near the zenith to low altitudes near the horizon. Thus the rays measured passed through longer and longer paths, according to the obliquity with which they crossed the atmospheric layers, and consequently they grew weaker and weaker as the sun declined lower and lower. From the spectro-bolometric measurements, standardized to calories by aid of the simultaneous pyrheliometer measurements and reduced to zero atmospheric absorption by the method of Langley we thus determined the intensity of solar radiation as it would be outside the atmosphere at mean solar distance. This has been called the solar constant of radiation. Its average value is about 1.93 calories per square centimeter per minute.

As I have said the results of 1903 at Washington indicated variations of this so-called constant over a range of nearly 10%. Owing to the prevalence of clouds at Washington all too few observations were available. Nevertheless when we compared such as we had with the anomalies of temperature of the North Temperate Zone as represented by meteorological observations at 89 stations, all regions showed a variation of temperature nearly parallel to, and of a proper magnitude to correspond with, the supposed variations of the sun.

In 1905 we transferred the observing to Mount Wilson, California, and with the exception of 1907 we have observed the 'solar constant,' in that relatively favorable place, usually from June to October of each year. The results have confirmed the apparent variability of the sun. It is impossible to go outside the atmosphere to observe, and we feared that the apparent variability of the sun might have been really due to defects in our estimation of the losses in the atmosphere. To check our work as far as possible we observed in 1908, 1909, and 1910 from the summit of Mount Whitney (4420 meters) the highest peak in the (older) United States. No error dependent

on altitude appeared. In 1914 we sent an automatic recording pyrheliometer by sounding balloon to 25,000 meters altitude. The result obtained was trustworthy and agreed with what we expected. All these checks confirmed the accuracy of our work and strengthened our belief in the solar variability.

Meanwhile in 1911 and 1912 I had observed in Algeria while my colleagues observed in California. Unfortunately for the proof of solar variability 1911 was cloudy and 1912 was the year the air was charged with volcanic dust by the great volcanic eruption of Mount Katmai, June 6, 1912. Nevertheless despite this handicap the results left little reasonable doubt that the sun is variable. High 'solar constant' values at Bassour, Algeria, coincided with high values at Mount Wilson, California, and vice versa, and equal increments of radiation were found at the two stations independently, notwithstanding that they are separated by one-third the earth's circumference.

In 1913 and subsequently the proof of solar variability was rendered impregnable. We investigated daily the distribution of radiation over the solar image formed by our tower telescope on Mount Wilson. The sun's image is, as you know, unequally bright at centre and edges, so that the curve of its intensity along a diameter takes the form of the letter U inverted. The steepness of the curves of the U varies with wave-length. But we found also that it varies from day to day, and that the variations are such that a greater contrast of brightness between center and edge occurs when the solar radiation as a whole is found to be diminished and vice versa. I suppose the outer layers of the sun vary in transparency. When more opaque they diminish the 'solar constant' but as the effect is greatest near the limb of the sun where the oblique path of the ray in the solar layers is greatest, the result is also to increase the contrast. At all events we have found a true variation of the sun, independent of the earth's atmosphere, which coincides in time of its fluctuations with the observed changes of the 'solar constant.'

Professor Pickering had kindly undertaken pyrheliometer measurements at Arequipa, Peru. These were carried on from 1912 to 1917. They tended to confirm in the more outstanding features the variation of the sun observed at Mount Wilson.

Dr. L. A. Bauer, whose remarkable campaign of magnetic observations lends lustre to the science of our country, has investigated certain minute disturbances of terrestrial magnetism for which no cause had been assigned. He finds them to be closely correlated with the variations of solar radiation we have observed.

Dr. H. H. Clayton, formerly of Blue Hill Meteorological Observatory, now of the Argentine Meterological Service, has investigated the variations of terrestrial atmospheric temperature and pressure for nearly fifty stations in all parts of the earth. He finds a close correlation of these meteorological variations with the irregular variations we have observed in solar radiation. Equatorial stations show a direct correlation in temperature. That is high solar radiation is followed by high temperature and vice versa. Many tem-

perate zone stations show opposition of variation. Polar stations show direction variation. If these results shall be confirmed and enlarged they bid fair to aid in actual weather forecasting, for the changes are by no means small.

In view of the scientific and utilitarian interest associated with the variability of the sun, I have long desired that several cloudless observing stations might take up 'solar constant' work. In 1914 I made a trip to Australia expecting that the Australian Government would take it up. This hope was frustrated by the war.

In 1916 Secretary Walcott appropriated from the income of the Hodgkins Fund to equip and maintain for several years such a station in South America, but owing to the war it was temporarily located in the North Carolina mountains in 1917. The station proved very cloudy, and now it has proved possible though very expensive to go to Chile.

After correspondence with the South African, Indian, Argentine and Chilean meteorological services I became convinced that near the nitrate desert of Chile is to be found the most cloudless region of the earth easily available. Dr. Walter Knoche of Santiago has most kindly furnished two years (1913 and 1914) of unpublished daily meteorological records for a number of Chilean stations. In his judgment and mine the best is Calama on the Loa River, Lat. S. 22° 28′, Long. W. 68° 56′, Altitude 2250 meters. For the two years the average number of wholly cloudless days is at 7 a.m. 228; 2 p.m., 206; 9 p.m., 299; and of wholly cloudy days, none. The precipitation is zero; wind seldom exceeds 3 on a scale of 12; temperature seldom falls below 0° or above 25°C.

Our expedition, Director Alfred F. Moore, Assistant Leonard H. Abbot, reached Calama June 25, 1918, equipped with a full spectro-bolometric, pyrheliometric, and meteorological outfit of apparatus, as well as with books, tools, household supplies, and everything which we could furnish to make the work successful and life bearable. We are under great obligations to the Chilean Government for facilitating the expedition in many ways, and to the Chile Exploration Company who have given the expedition quarters and observing station at an abandoned mine near Calama. Many others in Antofagasta, Chuquicamata and Calama have been of great assistance.

The apparatus is set up in an adobe building about 30 feet square, in which the observers also have sleeping apartments. A 15-inch two-mirror coelostat reflects the solar beam to the slit of the spectro-bolometer. We use a Jena ultra-violet crown glass prism and speculum metal mirrors in the spectroscope. The linear bolometer is in vacuum, and constructed in accord with complete theory for greatest efficiency. Its indications as measured by a highly sensitive galvanometer are recorded photographically on a moving plate which travels proportionally to the movement of the spectrum over the bolometer. Successive bolometric energy spectrum curves each occupying 8 minutes of time are taken from early morning till the sun is high and are

thus recorded on the plate. Their intensity indications at 40 spectrum positions are reduced by aid of a special slide rule plotting machine.

A pair of silver disk pyrheliometers is read simultaneously with each spectro-bolographic determination. Measurements of humidity, temperature, and barometric pressure accompany the bolometric observations. Also a pyranometer is employed to determine the total sky radiation.

The young men find pleasant companions at the great copper mine at Chuquicamata. At present they are boarding with a Chilean family at Calama, but as both are good cooks they may wish to board themselves. The railway and the river both pass the town of Calama, so that there is no such desert isolation as might be feared. To the east are the Andes Mountains. The peaks in that neighborhood rise to 16,000 or 17,000 feet. Some are volcanic but none of these are very near.

We hope the work will be continued for several years at least, and that nearly daily observations may be obtained. The application of the results to meteorology is something which may prove to have great possibilities. To exploit them we must first possess a long and nearly unbroken series of solar radiation observations.

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## *MAROON—A RECURRENT MUTATION IN DROSOPHILA*[1]

By Calvin B. Bridges

Marine Biological Laboratory, Woods Hole

Communicated August 23, 1918

The recessive eye-color 'maroon' was one of the early mutations in Drosophila (found March 13, 1912). This eye-color was found in a stock culture of wild flies, and was at first supposed to be a new appearance of the recessive mutation 'purple' discovered about a month earlier. Genetic tests (crosses between the two lines, etc.) showed that the new color was not the old purple, but was a new mutation almost identical with purple in appearance but entirely independent in origin and inheritance. This was the first of our now numerous cases of eye-color 'mimics,' in which two distinct genes produce practically the same somatic effect.

During the six months following the discovery of purple and maroon there were thirteen recorded appearances or 'purple' eye colors, which constituted our most striking 'mutating period' of 'epidemic of mutation.' Of these new 'purples,' tests showed that the first, fifth, sixth, and thirteenth were maroon, while the rest were true purple. A study of the pedigrees showed that these maroons had come from two independent occurrences of the maroon mutation, while the true purples likewise came from at least two separate acts of mutation. Maroon has reappeared independently on two subsequent occasions, so that this particular mutative process has occurred at least four times. The

same property of specific recurrent mutation has since been found to be characteristic of several other loci, of which notch, vermilion, rudimentary, and cut are the most striking examples.

At this time (April, 1912) pink was the only mutation known to be in the third chromosome (by definition) so that the $F_2$ results from the cross of maroon to pink constituted the only test as to whether maroon also is in the third chromosome. Maroon differs enough from pink in its characteristics so that the separation of the two forms could be accurate for about 85% of the individuals. The $F_1$ flies from the cross were wild-type (two non-allelomorphic recessives), and the $F_2$ flies were 485 wild-type: 215 maroon-like: 183 pink-like: 0 maroon pink.

It was expected that the flies which were homozygous for both maroon and pink would be as much lighter than pink as maroon is lighter than the wild-type, since this was the usual type of relationship. No flies lighter than pink were found, but it was uncertain whether this meant that the double recessive was actually absent because of strong linkage (no crossing over), or that the double form was present but indistinguishable from pink or from maroon. That the double form should be present, but not lighter than pink, would mean that maroon and pink are 'non-modifiers' of each other. Subsequent work has shown that 'non-modification' is the usual relation between the different pinks that have arisen.

That the apparent absence of the maroon pink class was a real absence due to linkage was soon proved: Tests, by means of $F_2$ and back cross results, of the linkage relations between maroon and the second chromosome mutant "arc" showed that these two genes assort with complete independence. The above tests showed that the locus of maroon was probably not in the second chromosome, and was therefore probably in the third chromosome. Meanwhile the body color ebony had arisen and had been shown to be linked to pink (Sturtevant, '13).[2] Since there could be no question of failure of identification of the double recessive maroon ebony the next step was to show that maroon is linked to ebony. The $F_2$ of the cross of maroon by ebony gave a 2 : 1 : 1 : 0 linkage ratio, which we had just learned to explain as the result of 'no crossing over in the male' (Morgan, '12).[3] The $F_2$ from the maroon by pink cross was thus proved to have been a 2 : 1 : 1 : 0 ratio, and was in fact the first such linkage result obtained for the third chromosome.

The location within the third chromosome of the gene for maroon was made easy and direct by the discovery and location of two excellent III chromosome dominants, 'dichaete' and 'hairless.' The locus of dichaete is quite near the left end of the third chromosome (at about 11.0) while that of hairless is near the middle (at about 32.0). The back cross tests showed that the locus of maroon is to the right of dichaete by about 4.2 units, or is at a position of 15.2 (11.0 + 4.2) when referred to the locus of sepia as the zero point. With hairless, maroon gave 21.2% of crossing over, which corresponds to the location of maroon between dichaete and hairless.

It was found that the chromosome in which the third independent mutation to maroon had occurred was carrying two other mutant genes: One of these, 'dwarf,' is a recessive that reduces the size of flies to only about half that of normal sibs; it also sterilizes females homozygous for it. The locus of dwarf is within a unit to the right of that of maroon. The other gene causes no observable somatic difference, but profoundly affects the process of crossing over in third chromosome in definite regions and by definite amounts. A distinct new system of cross-over values corresponds to the heterozygous and still another to the homozygous condition for the gene. A comparison of the behavior and effects of this gene with that of the already known genetic varition called 'C$_{III}$' showed that they were probably identical mutations. There was found to be 5.9% of crossing over between dichaete and maroon in femal s heterozygous for C$_{III}$, and 6.6% when females were homozygous for C$_{III}$. In the region near dichaete there is, as these values show, no great difference between the homozygous and heterozygous C$_{III}$ conditions.

[1] A full account of maroon with the detailed data on which the various statements of this paper are based will appear as a section of a publicatian by the Carnegie Institution of Washington.

[2] Sturtevant, *Science, New York*, N. S., **36**, 1913, (990–992).

[3] Morgan, *Ibid.*, **36**, 1912, (718–720).